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Preface

A survey of Heydar Radjavi

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Dedicated to Heydar Radjavi

"Heydar Radjavi is seventy years old? Impossible; he's too vigorous!" "He can't be seventy; he's too productive!" "Seventy? That can't be true; he's too good-looking!"

It is true; vigorous, productive and good-looking as he is, Heydar Radjavi is seventy years old as of January 17, 2005. Most of us slow down, at least a bit, as we enter our sixties. Not Heydar. As his list of publications establishes (see the end of this article for complete references to his research papers to date, followed by a list of his books), Heydar's productivity is an increasing function of his age.

As his many collaborators know, it is a great pleasure to work with Heydar. He is a very talented and knowledgeable mathematician. He loves thinking and talking about mathematics and working with others. His enthusiasm never seems to wane, even when numerous attacks on a problem fail, and even on those occasions when an unfillable gap is found in what the collaborating group had thought was a really nice discovery. He is helpful and pleasant to everyone, and is not at all competitive. Heydar is almost always in great spirits: the joy he finds in mathematics is part of his overall joy in life. He is one of the few people to whom the word "ebullient" is truly applicable. His lectures are wonderful: they are invariably very interesting and clear, and peppered with Heydar's special humor. He is, overall, the nicest kind of human being. It is a great pleasure to visit Heydar and his wife Ursula. In particular, they are great cooks: A nontrivial corollary of working with Heydar is the opportunity to sample the delicious Iranian meals he and Ursula prepare.

This survey consists of a brief look at Heydar's background followed by discussions of a few of the highlights of his mathematical work.

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Heydar was born on January 17, 1935 in the city of Tabriz in the Province of Azerbaijan in the country of Iran. The Azeris are a distinct ethnic group, about two thirds of whom live in Iran; the others live in what used to be called "Soviet Azerbaijan" (which includes the city of Baku). The native language of Azerbaijan is Azeri (a relative of Turkish); this was Heydar's mother tongue. When he entered primary school he began to learn Persian (also called Farsi), a very distinct language. He became so good at Persian that he seriously considered a literary career (one of his brothers became a well-known poet). Luckily for us, mathematics captured him in the last year of secondary school.

Even though it is his third language, Heydar's written and spoken English is excellent. His mathematical papers are always well-written, as a glance at any of the references will prove. He has also written a series of very interesting anecdotes from his childhood, as well as humorous pieces about life in North America.

Heydar left Tabriz for the first time at the age of 18, when he went to Teheran to attend college. At the time, the only university in Iran that offered a degree in mathematics was the University of Teheran. When he graduated at the top of his class in 1956, there was no research-level mathematics graduate program in Iran. Fortunately, however, Heydar got a reasonable background in undergraduate mathematics from Professors Ali Afzalipour, Taghi Fatemi, Mohammed Ali Nourghalitchi, and Manoutchehr Vessal, who then pointed him in the direction of graduate study at the University of Minnesota.

Heydar quickly adjusted to graduate school in a strange country. Under the supervision of Gerhard Kalisch, he wrote a very interesting master's thesis. Building on earlier work of J. Brenner [Acta Math. 86 (1951) 297–308], Heydar gave a complete set of unitary invariants for arbitrary matrices. This was the content of his first published paper [1]. He continued related research for his Ph.D., obtaining unitary invariants for arbitrary finite collections of compact operators (see [4]).

After completing his Ph.D. in 1962, Heydar spent a year as a visiting member of the Institute for Advanced Study in Princeton. Heydar felt a responsibility to return to Iran to help to educate students there; he was torn between that feeling of responsibility and his desire to exchange mathematical ideas and collaborate with others. (There was no email or fax in those days, and mail between Iran and North America was slow.) Over the next decade, Heydar oscillated back and forth. He became a tenured professor at Shiraz University in Iran, but spent a year (1964–1965) at the University of Illinois as assistant professor (they wanted to keep him) and several years (1967–1968, 1969–1970, 1972–1973) as visiting professor at the University of Toronto (they wanted to keep him too).

In 1973, Heydar accepted a two-year appointment as a Killam Senior Research Fellow at Dalhousie University in Halifax, Nova Scotia, Canada. This proved to be very fortuitous. Peter Fillmore, a close friend of Heydar's from the time when they were both graduate students at Minnesota, had earlier decided to return to his native Eastern Canada after having established himself at Indiana University. This time Heydar could not resist the offer of a permanent position that would be so conducive to his research, and he became professor of mathematics at Dalhousie. Over the last quarter of the twentieth century, Heydar Radjavi and Peter Fillmore made the small mathematics department at Dalhousie University into a large center for operator theory, attracting a number of colleagues, supervising many excellent Ph.D. students, and hosting several major conferences. As he got older and increasingly productive, Heydar determined that he would never retire. However, Dalhousie had mandatory retirement at age 65. In 1998, at age 63, Heydar precluded retirement by "quitting". Thus he is not a "retiree" (though he is certainly not a "quitter" either).

Since 1998, Heydar has been Professor Emeritus at Dalhousie, where he continues to supervise Ph.D. students and post-doctoral fellows and, occasionally, teach courses. He spent the academic years 2000–2001 and 2001–2002 as Visiting Professor at the University of New Hampshire (teaching graduate and undergraduate courses), and the spring of 2004 as Visiting Professor at the University of Waterloo (teaching a graduate course).

In addition to forming groups of collaborators, Heydar enjoys gathering broader semigroups of mathematicians together for gossip and chit-chat. Shortly after Heydar arrived in Dalhousie in 1973, he and Peter Fillmore started the tradition of "Tuesday lunches". Every Tuesday at 11:37 a.m. all members of the mathematics community who wish to attend meet in the lobby of the Chase Building and decide where to have lunch together. This tradition is so firmly established at Dalhousie that Tuesday lunches continue even when Heydar and Peter Fillmore are visiting elsewhere. Of course, Heydar also organizes Tuesday lunches at any department that he happens to be visiting for any period that includes a Tuesday. Heydar's contributions to Tuesday lunches have been immortalized in the poem by his Dalhousie colleague Bob Paré, written on December 5, 1989:

Tuesday Lunch 16th Year

HIM, HIM, HIM!

'Twas the day before Wednesday and all through the building,
Not a creature was stirring, the tension was building.
The hallways were empty, deserted and bare,
In the fear that Heydar soon would be there.

When down by the door there was such a clatter,
I ran to the stairwell to see what's the matter.
He was there with a flashlight and a tiny radar,
I knew in a moment it had to be Heydar.

He searched the department from top to bottom,
Grabbing Bruce Smith he shouted "I've got 'im".
But Bruce was busy, he had no time to lose,
Between probability and lunch, it was easy to choose.

Now Heydar was frantic as he checked the mailroom,
"Where's Dawson", he wondered, his face clouded with gloom.
Down the stairs, he came with a leap and a bound,
"More People", he yelled, "have got to be found!"
A few guys passed by on their way to Dalplex,
And Coley's excuse was that time is complex.

"Come Sastri, come Peter, come Keith, come Karl,
It's eleven thirty seven", he said with a snarl.
He turned in a huff and flew out the door,
And set off across campus with his group of four.

That he lose his good will after 16 years,
Was up there with all of my greatest fears.
But I heard him exclaim as they passed Howe Hall,
"Tuesday lunch, Tuesday lunch Tuesday lunch for all!"

Although he has been in North America for most of his career, Heydar has contributed enormously to the development of mathematics in Iran. Some of his influence has been through the contributions of students he taught in Shiraz, such as Mehdi Radjabalipour (still in Iran, in Kerman) and Ali Jafarian (now in New Haven after many years in Teheran). Also, he has maintained close contact with Radjabalipour, Jafarian and many other Iranian students and professors up until the present time, by correspondence and many visits.

Heydar and the Iranian graph theorist Mehdi Behzad spent the academic year 1966–1967 together in Shiraz. There was no one there with whom Heydar could collaborate on operator theory or linear algebra, so Heydar learned graph theory and worked with Behzad. Since Behzad wrote a paper with Paul Erdős, the resulting joint work (see Refs. [3,6,7,17]) gave Heydar the Erdős number of 2. Mehdi Behzad has been very active in the Iranian Mathematical Society, having served as its President for many years. He writes the following about Heydar's influence in Iran:

"Although Heydar spent only a small fraction of his productive life in Iran, his impact on the mathematical life of the country has been immense. He was one of the few who introduced functional analysis to Iranians. He was instrumental in the establishment of the Iranian Mathematical Society in 1971. The Society, which is now the most active scientific organization in the nation, started a new program in 2000 of electing honorary members. The first three to be elected were Heydar Radjavi of Dalhousie, F. Shahidi of Purdue, and C. Vafa of Harvard."

"Virtually all of the members of the Iranian mathematical community value Heydar for his thoughtfulness, dependability, charm, sense of humor, knowledge of Farsi, and, of course, for his mathematical talent. They are grateful to the editors of this journal for this special issue in his honor."

The third Iranian seminar on Linear Algebra and its Applications, to be held in Kerman, Iran in December 2004, will be in honor of the seventieth birthday of Heydar Radjavi.

In addition to producing so much mathematics, Heydar has produced two wonderful daughters, Marjan and Shirin (they join Heydar in the exclusive club whose members are known as “ebullient”).

Heydar has done a lot of fine mathematics—check out the references. In this survey we describe only a few of the many highlights of his research.

1. Self-commutators

Heydar’s first post-thesis published work [2] was a complete characterization of self-commutators, which are Hilbert space operators that can be written in the form $AA^* - A^*A$ where A is a bounded linear operator. Heydar proved that an operator can be written in this form if and only if it is Hermitian and its spectrum has at least one nonnegative limit point and at least one nonpositive limit point (which is equivalent to the statement that 0 is in the essential numerical range). This definitive result has become well-known and widely quoted.

2. Reflexive algebras

An algebra of bounded linear operators on a Banach space is said to be reflexive if it contains all the operators that leave invariant all the common invariant subspaces of the operators in the algebra. The first theorem on reflexive algebras was von Neumann’s double commutant theorem, which can be interpreted as saying that every von Neumann algebra is reflexive. The first result concerning non-self-adjoint reflexive algebras was Donald Sarason’s theorem [see Pacific J. Math. 17 (1966) 511–517] that weakly closed unital algebras of normal operators must be reflexive. At around the same time, William Arveson proved [see Duke Math. J. 34 (1967) 635–647] his density theorem: An algebra of operators is weakly dense if it is transitive and contains a m.a.s.a. (a maximal abelian self-adjoint algebra). This can be rephrased as stating that a weakly closed algebra containing a m.a.s.a. is reflexive if it is transitive.

Heydar’s first result on reflexive algebras, obtained in joint work with Peter Rosenthal [5,11], was a generalization of Arveson’s density theorem: A weakly closed algebra containing a m.a.s.a. is reflexive if its lattice of invariant subspaces is totally ordered. Radjavi and Rosenthal subsequently established [22] a different generalization of Arveson’s density theorem, that a weakly closed algebra containing a m.a.s.a. is reflexive if its lattice of invariant subspaces is orthogonally complemented. These results set the stage for Arveson’s deep theorem [Ann. Math. 100 (1974) 433–532] that a CSL algebra is reflexive if it has “finite width”. “Reflexive algebras” has become a standard topic in operator theory (see Ken Davidson’s book “Nest Algebras”, Longman, 1988 for a good overview), and has also stimulated the study of many other kinds of non-self-adjoint algebras.

3. Invariant subspaces

In addition to his work on reflexive algebras (described above) and on simultaneous triangularization (described below), much of Heydar's other research has involved invariant subspaces (including [14,16,18,20,24–27,29,35,43,46,53,56,64, 74,95]). Heydar's and Peter Rosenthal's book "Invariant Subspaces" (Springer-Verlag, Heidelberg, 1973; second ed., Dover, New York, 2003) contains a number of new results as well as many new treatments of previously known theorems. In particular, Chapter 6 includes extensions and unifications of earlier work obtaining existence of invariant subspaces by techniques of analytic continuation of local resolvents.

Victor Lomonosov's famous theorem (that an operator has a nontrivial invariant subspace if it commutes with an operator other than a multiple of the identity that in turn commutes with a compact operator other than 0) was discovered just in time to be presented in the first edition of "Invariant Subspaces". This beautiful result was a very broad extension of a sequence of invariant subspace theorems that had begun with the von Neumann–Aronszajn–Smith proof of the existence of nontrivial invariant subspaces for compact operators. In fact, the scope of Lomonosov's Theorem was not at all clear; it seemed quite possible that every operator on Hilbert space fulfilled its hypothesis. Almost a decade passed before Don Hadwin, Eric Nordgren, Heydar Radjavi and Peter Rosenthal were able to prove [48] that there are operators that do not satisfy Lomonosov's hypothesis.

Thus the invariant subspace problem, the question of whether every bounded linear operator on Hilbert space contains a nontrivial invariant subspace, remains open. Heydar, eternally optimistic, has bet Paul Halmos the price of a dozen eggs (attempting to make the bet inflation-proof) that the answer is affirmative.

4. Simultaneous triangularization of collections of matrices

In this section, we consider collections of linear transformations mapping a finite-dimensional complex vector space into itself. Such a collection is said to be simultaneously triangularizable (or, more simply, triangularizable) if there is a basis for the vector space such that the matrices with respect to that basis of every transformation in the collection are upper triangular.

The earliest result on this topic is Engel's classical theorem that a Lie algebra of nilpotent linear transformations is triangularizable. Jacobson ("Lie Algebras", Interscience, New York, 1962) found an interesting generalization: a set of nilpotent transformations is triangularizable if for each pair A, B in the set there is a scalar c such that $AB - cBA$ is in the set. Note that this does not require that the set be a linear space. In addition, Jacobson's Theorem includes not only Engel's Theorem but also the corresponding result for Jordan algebras. Heydar [71] extended this further, showing that a collection of nilpotent transformations is triangularizable if for each pair A, B in the set there is a transformation C in the unital algebra generated by A and B such that $AB - CA$ is in the collection. Another nice result that Heydar included in the same paper states that a set of transformations closed under the Lie product is triangularizable if and only if all the Lie products of transformations in the collection are nilpotent.

A fundamental theorem required for the study of triangularization is Burnside's Theorem that the only transitive algebra of linear transformations on a finite dimensional space is the algebra of all linear transformations on the space. Heydar and collaborators Luzius Grünenfelder and Matjaž Omladič proved [85] an analogue for Jordan algebras: a transitive Jordan algebra is either the algebra of all transformations or is similar to the Jordan algebra of symmetric matrices.

An interesting sequence of theorems on triangularizability of semigroups of linear transformations culminated in Heydar's beautiful and definitive theorem. Levitzki [Math. Ann. 105 (1931) 620–627] proved that a semigroup of nilpotent linear transformations is triangularizable. Kolchin [Ann. Math. 49 (1948) 774–789] proved that a semigroup of unipotent matrices (i.e., matrices whose only eigenvalue is 1) is triangularizable. Kaplansky ("Contributions to Algebra" edited by Bass, Cassidy and Kovaciks, Acadmic Press, New York, 1977, pp. 233–237) found a very nice common generalization of the theorems of Levitzki and Kolchin: a semigroup with constant trace is triangularizable. However, even Kaplansky's sufficient condition is obviously not necessary for triangularizability. Heydar [65] found the ultimate result: a semigroup of linear transformations is triangularizable if and only if the trace is permutable on the semigroup, in the sense that the trace of ABC is the same as the trace of BAC for all A, B and C in the semigroup.

5. Simultaneous triangularization of collections of operators

There is a coordinate-free definition of triangularizability that applies to operators on Banach spaces of any dimension. A collection of bounded linear transformations is said to be triangularizable if there is a chain of subspaces, maximal as a subspace chain, every element of which is invariant under every operator in the collection. It is easy to see that, in the finite-dimensional case, this reduces to the definition given above.

The first result on simultaneous triangularizability for collections of operators on infinite dimensional spaces was Wojtynski's generalization of Engel's theorem to Lie algebras of Hilbert–Schmidt operators [Bull. Acad. Polon. Sci. Ser. Sci. Math. 24 (1976) 797–801]. In the years 1980–2000, Heydar and his co-workers and several other mathematicians extended virtually all of the finite-dimensional results to collections of compact operators. A crucial contribution was made by Turovskii [J. Funct. Anal. 162 (1999) 313–322] who extended Lomonosov's work to the beautiful result that a semigroup of compact quasinilpotent operators must be triangularizable. Moreover, an example found by Heydar and collaborators [66] shows that almost none of the classical results extend to collections of arbitrary bounded operators, so the infinite dimensional situation is now well understood.

A fairly complete treatment of the work on triangularization in both the finite and infinite dimensional cases is contained in the book “Simultaneous Triangularization” by Heydar Radjavi and Peter Rosenthal (Springer, New York, 2000). Fortunately, Turovskii, like Lomonsov a quarter of a century earlier, came along just in time for inclusion in this book. Unfortunately, however, a result of Shulman and Turovskii [J. Funct. Anal. 177 (2000) 383–441] was a little too late. Shulman and Turovskii extended Wojtynski’s theorem to the ultimate generalization of Engel’s theorem: A Lie algebra of compact quasinilpotent operators is triangularizable.

6. Structure of matrices

Heydar loves matrices, and loves to think about their structure. He has characterized those matrices that can be written as products of involutions [30,32,51] and as products of Hermitian matrices and symmetries [9], and has studied co-squares [49]. Heydar’s affinity for matrices has played a strong role in much of his research.

7. Other results on semigroups

In recent years, Heydar has done a lot of work on semigroups of matrices and of operators. In joint work with Matjaž Omladič [94], Heydar described the structure of irreducible semigroups of matrices with spectral radius 1. Other results on irreducible semigroups of matrices include the study of semigroups of matrices each of which has eigenvalue 1 [128]. Heydar et al. obtained various results concerning transitive linear semigroups [111,113].

A band is a semigroup of idempotent matrices or operators. Heydar and collaborators have studied the structure of principal ideal bands [104] and reducibility of bands and band algebras [112,116].

Heydar has done research on semigroups of matrices and operators that are nonnegative in the sense that they preserve the collection of nonnegative vectors or functions. A collection of matrices is said to be decomposable if they all leave invariant a fixed span of some nontrivial subset of the basis vectors, and to be completely decomposable if it is triangularizable by a chain of such subspaces. Theorem 5.1.6 of “Simultaneous Triangularization” gives a number of characterizations of completely decomposable semigroups of nonsingular nonnegative matrices, including the fact that it suffices that each individual member of the semigroup be completely decomposable. Infinite dimensional generalizations of this theorem have been found by Heydar and Gordon MacDonald [134]. The question of when reducible semigroups of nonnegative operators are decomposable is investigated by Livshits, MacDonald, Mathes and Radjavi [116]. Heydar has extended the well-known Perron–Frobenius

Theorem to the context of semigroups of matrices and compact operators ([106] and "Simultaneous Triangularization").

To learn more about Heydar's contributions to mathematics, see any of the following references.

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